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ABSTRACTS

This section consists of the abstracts of papers presented to the JACOB MARSCHAK INTERDISCIPLINARY COLLOQUIUM ON MATHEMATICS IN THE BEHAVIORAL SCIENCES at the Graduate School of Management, University of California, Los Angeles, California 90024.

The readers who are interested in a particular article should write to the author for the entirety.

Document technology. Robert Spinrad, *Corporate Strategy Office, Xerox Corporation, Palo Alto, CA 94304, U.S.A.*

The concern is with documents – many, many documents, interlinked and cross-references, having change-histories and chronologies, and forming a body of information whose structure, itself, carries meaning. A 'document' can be any combination of text, diagram, sound and image, static or dynamic... the expressive vehicle for a body of work. The onrush of new technology is changing the classical notion of a document, as a set of bound pages on which words and pictures are imprinted, to one allowing a far greater diversity of content and medium. Certainly, electronic storage and transient, on-screen displays can coexist with paper versions. More importantly, the restricted, linear structure of current documents is changing, allowing for multiple organizational patterns of the material, some of it animated. Zooming in for greater detail, and out for context, is enabled. Significantly, the electronic document now embeds active processes. Change to one document triggers actions affecting other, related documents. Voice annotation is a useful option. Multiple layers of commentary by different people are enabled as in the Talmudic tradition of religious scholarship.

Cyclic tournaments and cooperative majority voting: A solution. Thomas Schwartz, *Department of Political Science, UCLA, Los Angeles, CA 90024, U.S.A.*

Imagine a committee, governed by majority rule, that chooses cooperatively from finite, nonempty subsets (A, B, etc.) of a set X of alternatives. For a given A, the *solution* to the voting game is the set of alternatives in A – call it $C(A)$ – that a majority might make a contract to support. Identifying $C(A)$ is a notorious problem when the relation P of majority preference is cyclic (e.g., one member of a three-

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man committee prefers x to y to z, another y to z to x, and the third z to x to y, so that $xPyPzPx$). The problem is often simplified by assuming, as I do here, that P is a *tournament* on X (an asymmetric binary relation on X that is connected in X). Given this assumption, a growing literature has focused on three potential solutions: the *top-cycle set*, a comparatively large set that can have Pareto-inefficient elements; the *uncovered set*, a Pareto-efficient subset of the top cycle set; and the *Banks set*, a subset of the uncovered set. Each of these sets is perforce nonempty, and each is definable in terms of P. Other familiar cooperative-solution concepts (the core, the V-solution, the bargaining set, etc.) lack the former trait and sometimes the latter. The difficulty we face in evaluating all such concepts is that parliamentary procedure does not structure cooperative voting as it does noncooperative voting. The final outcome of noncooperative voting, sincere or strategic, can be defined in terms of an extensive form (or simpler arboreal structure) determined by the agenda. But under cooperative voting, the agenda is a mere formality: a majority make a contract to support a particular alternative before voting begins.

In place of the invisible extensive form, I offer three axioms and a characterization theorem. To motivate the axioms, interpret the choice function C so that $C(A)$ comprises those alternatives which a majority might make a *final contract* to choose from A, not merely a tentative contract that gives way to another and perhaps another in a preliminary process of recontracting. Were such a process to begin, it would end with the choice of a Condorcet winner – a member of A that bore P to every other – if one existed. Otherwise it would continue indefinitely and end somewhat arbitrarily, but the final choice – the choice stipulated by the final contract – would have to belong to $C(A)$, and there is no smaller set to which it would have to belong.

Suppose $x \in A$ and yPx for some $y \in A$. Then any tentative contract to choose x from A can be upset by a contract to choose something else. What else? Not necessarily y. For y might be one of several alternatives in A, say $y_1, \ldots, y_k$, that bear P to x. Could just any one of these y_i replace x in the imagined recontracting process? To suppose so is to suppose, in effect, that a possible replacement for x is selected in some more or less arbitrary manner and pitted against x in a contest in which each committee member votes for the alternative he prefers. But such a procedure would be noncooperative. Because members act cooperatively, they will *cooperatively choose a replacement for* x: some majority will make a contract to replace x with one of the y_i, a contract that is final in the process of replacing x, though perhaps only tentative in the larger process of choosing from A. Given our inerpretation of C, this means that y_i can replace x in the recontracting process iff $y_i \in C(\{y_1, \ldots, y_k\})$. In other words, a tentative contract to choose x from A can be upset by a contract to choose y iff $y \in C(\{z \in A \mid zPx\})$.

Call a subset B of A *C-retentive in A* iff no tentative contract to choose an element of B can be upset by any tentative contract to choose an element of $A - B$ ($y \in C(\{z \in A \mid zPx\})$ for no x in B and y in $A - B$). Then the recontracting process, having entered or begun within a C-retentive subset, can never depart. But it *can* depart from any *non*-retentive subset. So unless $C(A)$ were C-retentive, we could not predict (as we wish to do) that the final choice from A will belong to $C(A)$. Hence our first axiom:

> RETENTIVENESS $C(A)$ is C-retentive in A.

What we expect of $C(A)$ is not only that the recontracting process would never depart from $C(A)$ but that $C(A)$ have no proper subset in which the process must end up, else our prediction that the final choice will belong to $C(A)$ would not be the strongest we could make. So C should also satisfy:

> PREDICTIVE STRENGTH There is no proper subset B of $C(A)$ such that B is C-retentive in A but $C(A)-B$ is not.

The recontracting process, if it began (by chance or design) in a C-retentive subset, would never depart, so the final outcome would belong to that subset. Therefore, C must satisfy our last axiom:

> INCLUSIVENESS If B is C-retentive in A then some member of B belongs to $C(A)$.

The promised characterization theorem is that there exists a unique choice function satisfying the three axioms. That function is *TEQ*. *TEQ*(A), the *tournament-equilibrium set* corresponding to A, is always a subset, sometimes proper, of the top-cycle, uncovered, and Banks sets corresponding to A.

Multivariate analysis with optimal scaling of the variables. Jan de Leeuw, *Social Statistics Program UCLA, 9240 Buhnche Hall, 405 Hilgard Avenue, Los Angeles, CA 90024, U.S.A.*

In traditional multivariate analysis relationships between variables are investigated. Variables $y_1, \ldots, y_m$ are elements of a linear space $\mathscr{L}$, in which typically an inner product $\langle \cdot, \cdot \rangle$ is defined. The inner product of two variables is usually called their *covariance*, the squared length of a variable is its *variance*. In many cases variables are random variables, i.e. measurable functions defined on a probability space, the inner product can be used if we assume that the variables have finite variance.

We argue that in the social sciences there are many cases in which it is more appropriate to model variables as *subsets* $\mathscr{L}_j$ of $\mathscr{L}$. Because we want to continue to use the linear structure of the space, we assume throughout that the $\mathscr{L}_j$ are linear *subspaces* (although most of the theory readily extends to convex cones). In the particular form of nonlinear multivariate analysis we describe here, we study relationships of the m subspaces $\mathscr{L}_j$.

In traditional multivariate analysis techniques typical study *aspects* of covariance matrices. We look at the first few eigenvalues, the multiple correlation(s), the deter-